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SUMMARY

It is shown that the control force of ordinary ailerons may be reduced to zero over a range of deflections and at a given flight condition by the use of an appropriate differential movement. Approximations to the ideal motion obtainable with a simple linkage are discussed and a chart that enables the selection of an appropriate crank arrangement is presented. Various aspects of the practical application of the system are discussed and it is concluded that a small fixed tab, deflected to trim both ailerons upward, would be advantageous.

INTRODUCTION

One of the most exacting requirements of a lateral-control system is the provision of an adequate degree of control with a small expenditure of operating effort. It appears that a differential linkage can, when properly designed, be a very effective means of reducing the operating force of ordinary ailerons. Several other advantages accrue to the differential and such systems are widely used. The possible reduction of control force appears to be of primary importance, however, and it is therefore of interest to discuss some rules for the design of a linkage that will afford the greatest advantage in this respect.

The reduction of operating force with a differential linkage is accomplished by taking advantage of the up-floating tendency of the ailerons. This floating tendency is apparent in measurements of aileron hinge moments, which generally show an offset moment (C_{h_0}) at the neutral setting. (See fig. 1.) This moment varies with angle of attack and with airfoil profile.

With a differential linkage the ailerons on opposite tips of the wing begin to move at different rates immediately after they are deflected from neutral, the downgoing aileron moving more slowly than the upgoing one. Thus with the ailerons deflected the upward pressure on the upgoing aileron, which tends to increase the deflection, has a greater mechanical advantage at the control stick than does the upward pressure of the downgoing aileron. The combination of a reduced upward pressure and an increased mechanical advantage of the upgoing aileron tends to nullify the effect of the increased upward pressure and reduced mechanical advantage of the downgoing aileron to the extent that within certain limits the operating force may be reduced or even reversed.

If the ailerons are connected to the control stick with nondifferential gearing, the effect of the initial hinge moment at the neutral setting is not felt in the stick force required to deflect them. In this case the mechanical advantage of one aileron with respect to the other remains the same, so that the initial offset moment (C_{h_o}) on one aileron is exactly balanced by that on the other. The only force experienced at the stick is that due to the difference of the aerodynamic hinge moments of the two ailerons brought about by their deflection.

DEFINITIONS OF SYMBOLS

c_w, chord of wing.

c_a, chord of aileron.

c_t, chord of tab.

C_h, aileron hinge-moment coefficient.

C_{h_o}, aileron hinge-moment coefficient at zero deflection (normally negative).

C_{h_s}, resultant hinge-moment coefficient acting at control stick (negative when moment opposes deflection).

C_l, rolling-moment coefficient.

C_n, yawing-moment coefficient.

C_L, lift coefficient.

φ_1, angle of bank 1 second after deflection of aileron control.

δ_N, angular setting of aileron crank when neutral.

θ_N, angular setting of control-stick crank when neutral.

$\Delta\theta$, angular movement of control-stick crank.

δ_u, upward deflection of aileron

δ_d, downward deflection of aileron

δ_{uf}, upfloating angle of aileron

δ_{td}, downward deflection of tab

(δ_u, δ_d, δ_{uf}, δ_{td}) Taken as positive numbers.

CALCULATION OF CONTROL FORCE AND WORK OF DEFLECTION

A calculation of the effective moment coefficient acting at the control stick will show how the characteristics of the differential linkage affect the operating force. The hinge-moment coefficient of an ordinary aileron may be calculated with sufficient accuracy by the formula (neglecting weight of aileron),

$$C_h = C_{h_o} + \delta \frac{dC_h}{d\delta} \tag{1}$$

This formula applies to a single aileron. The effective moment coefficient acting at the control stick or wheel due to the up aileron is

$$C_{h_s} = \left(\delta_u \frac{dC_h}{d\delta} - C_{h_o}\right) \frac{d\delta_u}{d\theta} \tag{2}$$

and that due to the down aileron

$$C_{h_s} = \left(\delta_d \frac{dC_h}{d\delta} + C_{h_o}\right) \frac{d\delta_d}{d\theta} \tag{3}$$

The rates of change of the up and the down aileron angles throughout the range of the stick deflection are determined by the characteristics of the particular differential linkage used.

The various terms of equations (2) and (3) may be so collected as to represent two components of the total moment, one tending to return the control to neutral and the other tending to displace the control away from neutral. Thus,

$$C_{h_s} = - C_{h_o}\left[\frac{d\delta u}{d\theta} - \frac{d\delta d}{d\theta}\right] + \frac{dC_h}{d\delta}\left[\delta_u \frac{d\delta u}{d\theta} + \delta_d \frac{d\delta d}{d\theta}\right] \qquad (4)$$

The term containing C_{h_o} represents the reduction of C_{h_s} due to the difference of mechanical advantages.

With regard to the reduction of operating force, it is seen that the problem is to secure the proper relationship between the two components of equation (4). A large upfloating angle (indicated by large C_{h_o}) and a rapidly increasing difference between the mechanical advantages of the two ailerons make for the displacing tendency, whereas a large slope of the hinge-moment curve and a large average mechanical advantage of both ailerons (large $d\delta/d\theta$) make for a large restoring force. If the control stick is to tend to return to neutral when displaced, C_{h_s} must be at least slightly negative.

An examination of the hinge-moment curve (fig. 1) will show that the balancing effect of the differential can be simply described in terms of the work of deflecting the ailerons. Work is gained by allowing the aileron to rise. The resultant work is found by deducting that exerted on the down side from that gained on the up side. The two components are represented by the areas under the hinge-moment curve on either side of neutral. The formula for the resultant work is

$$C_{h_o}\ \delta_d + \frac{dC_h}{d\delta}\frac{\delta d^2}{2} - C_{h_o}\ \delta_u + \frac{dC_h}{d\delta}\frac{\delta u^2}{2} \qquad (5)$$

If the work of deflection is made zero at every point, the stick force (which may be calculated as the slope of the curve of work against deflection) will also be zero at every point. Hence an idealized differential motion of the ailerons that gives complete balance may be calculated by means of this expression for the work of deflection. By equating the work to zero and rearranging

$$\delta_d = \frac{-C_{h_o} \pm \sqrt{C_{h_o}{}^2 - \frac{dC_h}{d\delta}\left[\frac{dC_h}{d\delta}\ \delta u^2 - 2C_{h_o}\ \delta u\right]}}{\frac{dC_h}{d\delta}} \qquad (6)$$

If C_{h_o} and $dC_h/d\delta$ are known, this formula may be used to calculate the simultaneous upward and downward positions of the ailerons for which the work of deflection is zero. A differential linkage arranged to give these simultaneous positions of the ailerons would thus require no operating effort.

A decidedly simpler formula than (6) results if the aileron characteristics C_{h_o} and $dC_h/d\delta$ are expressed in terms of the upfloating angle

$$C_{h_o} \frac{d\delta}{dC_h} = \delta_{uf} \tag{7}$$

(See fig. 1.) The resultant formula is

$$\delta_d = \sqrt{(\delta_{uf} + \delta_u)^2 - 2\delta_u^2} - \delta_{uf} \tag{8}$$

The ratio of the rates of travel of the two ailerons is simply

$$\frac{d\delta_d}{d\delta_u} = \frac{(\delta_{uf} - \delta_u)}{(\delta_{uf} + \delta_d)} \tag{9}$$

(Note that δ_{uf}, δ_u, and δ_d are taken as positive numbers.) Curves of such idealized differential motions for ailerons having different floating angles are shown in figure 2. It is probable that a number of more or less complicated mechanical linkages that would give the ailerons motions approximating these curves could be devised. The ordinary simple linkage consisting of two properly set cranks connected by a rod is of most interest, however, and the following discussion is devoted chiefly to the problem of approximating the limiting degree of balance with such a simple arrangement.

METHOD OF APPROXIMATING LIMITING DEGREE OF BALANCE

The preceding discussion led to the determination of curves of aileron deflection giving zero stick force derived without reference to the limitations of mechanical linkages. Although the discussion was confined to ailerons showing a straight-line hinge-moment variation, the procedure of deriving a curve similar to those of figure 2

for the case of a more irregular hinge-moment variation will be obvious. Once the limiting curve showing the deflections that result in zero operating force is determined, it becomes necessary to devise a geometrical arrangement of levers that will approximate this motion.

Although it is not thought desirable completely to eliminate the control force at any flight condition, it is useful to consider this condition as a limiting adjustment of the differential. In a given case the stick force can be balanced out at only one angle of attack and, since the floating angle becomes smaller, the effectiveness of the balancing diminishes as the angle of attack is reduced. Hence if the stick force is made to become zero at an angle of attack higher than normally encountered in flight, overbalance of the control in normal flight will be guarded against and, at the same time, the greatest permissible reduction of control force will be approached.

The ideal curves given in figure 2 cannot, of course, be exactly reproduced by a simple mechanical linkage. Figure 3 illustrates the simplest type of linkage used in practice. Movement of the control causes the stick cranks to move oppositely through equal angles $\Delta\theta$ from their neutral positions. The diagram shows the downgoing crank in the dead-center position. Figure 4 illustrates the computation of the mechanical characteristics of such a simple linkage. Here it is assumed that the lengths of the various levers areknown. The formula then gives δ in terms of θ.

Such linkages can be adjusted to give aileron movements similar to those shown in figure 2 and can, in fact, be made to satisfy as many as four conditions in approaching such a curve, since four independent adjustments of the linkage may be made. The reduction of the stick force to zero at four points would, however, eliminate all possible linkages but one and would require a definite spacing of the crank centers and definite radii of the cranks, as well as specific neutral settings.

When trying to approximate the ideal differential motion by means of a simple mechanical linkage, it is not feasible to satisfy all the possible conditions. If only two minimizing conditions are imposed on the stick-force curve, it may be ascertained that the differential chosen is reasonably near the limiting one and, at the same time, an arbitrary choice of two of the geometric parameters of

the linkage may be made. The spacing of the crank centers and the relative radii of the two cranks may therefore be left to be dictated by other considerations.

Figure 5 shows a type of stick-force curve that satisfies two very simple criterions. First, the slope of the curve is zero at the beginning of the deflection; and, second, the force is zero at a deflection of the up aileron equal to the floating angle. The last criterion is satisfied by arranging for the downgoing aileron to reach dead center at this point or, algebraically expressed,

$$(\delta_u)_{\delta_d=\delta_{d_{max}}} = \delta_{uf} \qquad (10)$$

In order to show how the first of the two criterions may be satisfied, it will be necessary to calculate the slope of the stick-force coefficient curve at zero deflection $(\theta = \theta_N)$. If there is an infinitesimal displacement of the control from neutral, the difference of the mechanical advantages of the two ailerons will be

$$2d\theta \frac{d^2\delta}{d\theta^2} \qquad (11)$$

This difference multiplied by the initial or offset hinge moment (C_{h_o}) will give the infinitesimal displacing moment at the start of the deflection. The changes of aerodynamic hinge moment, $d\delta_u \frac{dC_h}{d\delta}$ and $d\delta_d \frac{dC_h}{d\delta}$, do not contribute to this quantity. The restoring or stabilizing tendency for infinitesimal deflection is simply

$$2d\theta \frac{d\delta}{d\theta} \frac{dC_h}{d\delta} \frac{d\delta}{d\theta} \qquad (12)$$

Here the infinitesimal changes of mechanical advantages play no part. The starting slope of the curve is then

$$\left(\frac{dC_{h_s}}{d\theta}\right)_{\theta=\theta_N} = 2\left[\frac{dC_h}{d\delta}\left(\frac{d\delta}{d\theta}\right)^2 - C_{h_o}\frac{d^2\delta}{d\theta^2}\right] \qquad (13)$$

Since $C_{h_o} = \delta_{uf}\frac{dC_h}{d\delta}$ this result may be expressed

$$\left(\frac{dC_{h_s}}{d\theta}\right)_{\theta=\theta_N} = 2\frac{dC_h}{d\delta}\left[\left(\frac{d\delta}{d\theta}\right)^2 - \delta_{uf}\frac{d^2\delta}{d\theta^2}\right]_{\theta=\theta_N} \qquad (14)$$

The condition for zero initial slope is then simply

$$\left(\frac{d\delta}{d\theta}\right)^2_{\theta=\theta_N} = \delta_{uf}\left(\frac{d^2\delta}{d\theta^2}\right)_{\theta=\theta_N}$$

(Note that δ_{uf} must now be expressed in radian measure.)

CHARTS FOR SELECTION OF LIMITING DIFFERENTIAL

It will be noted that the only characteristic of the ailerons appearing in either of the two criterions is the floating angle δ_{uf} (neglecting the implied assumption of a straight-line hinge-moment variation). It thus appears that the choice of differential as defined by these two criterions depends only on the floating angle. With the essential aileron characteristics thus limited, it was found feasible to make a series of calculations that would show the adjustment of a differential necessary to satisfy both criterions for a minimum stick force.

Figure 6 shows the results of such a series of calculations. This chart shows directly the angular settings of stick and aileron cranks to be used for a given up-floating angle at several spacings of the crank centers. It was assumed that the cranks were of equal radius. The maximum down-aileron deflection is shown in each case and it is to be noted that, if the maximum deflection of the upgoing aileron exceeds the floating angle, the down aileron will pass beyond dead center and return toward neutral. Since a differential selected by means of these charts will give what amounts to complete balance at the flight condition corresponding to the assumed floating angle, it is essential that this angle be at least as large as the maximum encountered in flight, which, as might be expected, usually occurs at or beyond maximum lift.

IMPROVEMENT OF BALANCE AT LOW ANGLES OF ATTACK

BY MEANS OF A FIXED TAB

It is evident that the same degree of balance cannot be attained at all flight speeds with any type of differential, inasmuch as the floating angle varies. At higher speeds the degree of balance becomes less and the control

force correspondingly greater. Calculations have shown, however, that a considerable advantage usually accrues to the differential system even at the highest flight speeds.

Although the possibility of securing complete balance at any one flight condition does not depend to a great extent on the characteristics of the aileron, modification of these characteristics can be very effective in improving the balance over a range of flight conditions. Thus, an aileron that shows only a small variation of floating angle over the flight range will be nearly ideally balanced under all conditions.

Wing-section theory indicates that the floating tendency of a flap may be characterized by two effects, namely:

1. A constant floating tendency due to camber of the airfoil and influenced mainly by the degree of camber near the flap trailing edge. This effect varies with the flap chord and is measured by the floating angle at zero lift of the airfoil.

2. A floating tendency varying with angle of attack of the wing section. This effect is the same for all airfoil shapes but varies with flap chord.

It is the latter tendency that is significant in causing the undesirable increase in stick force as the angle of attack is reduced. In general, the variation of floating angle with angle of attack can be reduced by reducing the chord of the aileron. This procedure, however, reduces the maximum floating angle in proportion so that the change in percentage with angle of attack remains about the same. Wind-tunnel tests show that a large constant floating effect can be produced by a relatively small camber of the trailing edge of the aileron (e.g., by a tab). The most nearly ideal arrangement for balance would thus incorporate a bent trailing-edge tab with an aileron of small chord.

Figure 7 shows the variation of floating angle with flap chord and angle of attack. The angles shown were computed by finding the moments of the pressure acting on rear portions of Clark Y and N.A.C.A. 23012 wing sections (reference 1 and unpublished data). This procedure gave the hinge moments at zero deflection and the floating angles were computed therefrom by using an empirical value of the slope of the hinge-moment curve $\left(\frac{dC_h}{d\delta} = -0.0085\right)$.

Figure 8 shows results of experiments (reported in reference 2) with a 2.5-percent-c_w tab on a 25-percent-c_w aileron. Deflecting the tab 10° had the effect of nearly doubling the maximum floating angle. The undesirable variation of floating angle with angle of attack, expressed in terms of percentage of the maximum, decreased accordingly. Figure 9 summarizes the results of some experiments made with tabs in the N.A.C.A. 7- by 10-foot wind tunnel.

CALCULATED EXAMPLE

Some calculations have been made to illustrate the application of the principles discussed. The results are summarized in figure 10, which shows the reduction of operating force that can be attained with a suitable differential both with and without a fixed tab. The chart has as ordinate the resultant moment coefficient acting at the control-stick crank divided by the lift coefficient $-\Sigma C_h/C_L$. This quantity is taken as a measure of the operating force. Division by the lift coefficient is made to take account of the increase in dynamic pressure corresponding to a reduction in angle of attack of steady flight. The abscissa represents a measure of the deflection of the control and is the computed angle of bank that a small average airplane (1,600 pounds) would attain in 1 second after the instantaneous partial deflections of the ailerons thus indicated. Such a conversion was necessary in order to compare equal up-and-down deflections with various degrees of differential movement of the ailerons on an impartial basis. Such deflections are thus measured by the banking effect they produce. The computation of banking effect is given in reference 3 by a simple formula,

$$\varphi_1 = \left(\frac{\partial\varphi_1}{\partial C_l}\right) C_l + \left(\frac{\partial\varphi_1}{\partial C_n}\right) C_n \qquad (16)$$

where $\left(\frac{\partial\varphi_1}{\partial C_l}\right)$, etc., are constants for a given airplane at a given flight speed. The curves of aileron hinge moment given in figure 8 were used and data on the rolling- and yawing-moment coefficients were taken from reference 4. As no limit was set on the maximum deflection of the control and no gearing ratio of the control stick to the stick crank of the differential was assumed, the values given are only comparative. The maximum degree of control

usually shown by airplanes of this size corresponds to $\varphi_1 = 20^\circ$ or 25° at a lift coefficient of 1.0.

The top pair of curves of figure 10 was computed for equal up-and-down ailerons without balance of any kind. The middle pair of curves shows the degree of balance that can be attained by a differential linkage without modification of the aileron floating characteristics. The differential linkage was selected with the aid of the chart (fig. 6). A crank spacing of four times the crank radius ($R = 1/4$) was assumed. The floating angles of the aileron without the tab are indicated in figure 8. Since the maximum floating angle in this condition was only 12°, the downward deflection of the ailerons was limited to slightly under 5°. (See fig. 6.) Thus a reversal of the motion of the down aileron occurred at this angle. Further deflection of the system then gave reduced control effectiveness and resulted in the sharp upward sloping of the stick-force curve ($C_L = 1.0$) that is apparent near $\varphi_1 = 22^\circ$. It appears that, in general, the best results will be obtained when the maximum deflection permitted does not greatly exceed this reversal point.

The bottom pair of curves of figure 10 gives an indication of the remarkable effect of a small fixed tab. The tab and deflection assumed (2.5 percent wing chord, down 10°) would give the trailing edge of a 5-foot-chord wing a downward displacement of only 1/4 inch. This modification served to increase the maximum floating angle of the ailerons from 12° to 20°, thereby permitting the use of a differential with a greater maximum downward deflection. It will be noted, in addition, that the stick force required for control at high speed ($C_L = 0.35$) is much less, and is also more nearly coincident with the force required at low speed ($C_L = 1.0$), than was the case with the unmodified ailerons. The beneficial effect of a fixed tab would be expected to be even more apparent in the case of narrow-chord ailerons.

Inasmuch as the floating angles corresponding to 15° angle of attack (fig. 8) were used in the selection of the differentials, the control will begin to show overbalance at this angle of attack. The form of the curves of stick force against deflection will be similar to those given but the curve will lie more nearly along the axis.

One difficulty that might arise in practice was brought out in the sample computation given. Here, in the

case of the aileron without the tab, the maximum upfloating angle was less than the maximum angle of deflection that might be required for control. The down aileron will, in this case, reverse its motion before the maximum deflection of the control stick is reached. It was observed in the computation that the stick-force curve rose sharply after this deflection was reached, showing that further deflection of the system was inefficient. The differential selected on the basis of the given assumptions was such as to impose a minimizing condition at this deflection. It is evident that a slightly different linkage might give better results at higher deflections; hence it might have been better to have chosen a different criterion. In such a case it would be advisable to make several trial computations, assuming fictitious floating angles higher than the actual angle. The desirability of keeping the maximum deflection of the stick cranks low should be especially emphasized. Since the maximum angular travel of the control stick or wheel is naturally limited, a large deflection of the stick crank of the differential means that the pilot will have to operate the system at a large mechanical disadvantage. When a deflection such that the down aileron reverses its motion is reached, further deflection does not cause the control rolling moment to increase very rapidly; hence a relatively larger maximum deflection will be needed for the requisite amount of control than would be the case if the reversal did not occur.

Wind-tunnel experiments show that the floating angles of ailerons are considerably influenced by insignificant details of construction. It is difficult, for instance, to establish any definite relation between the floating angle and the chord of a flap from small-scale wind-tunnel observations. The presence of a gap between the aileron and wing affects the floating angle and is also known to be decidedly detrimental to control. Furthermore, motions of the airplane such as rolling or sideslipping affect the pressure on the ailerons and thereby change the action of the differential to some extent. In view of these considerations, it would seem advisable to incorporate in differential ailerons either an adjustable tab or a deformable trailing edge so that unpredictable defects of the system may be remedied during trial.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., November 13, 1936.

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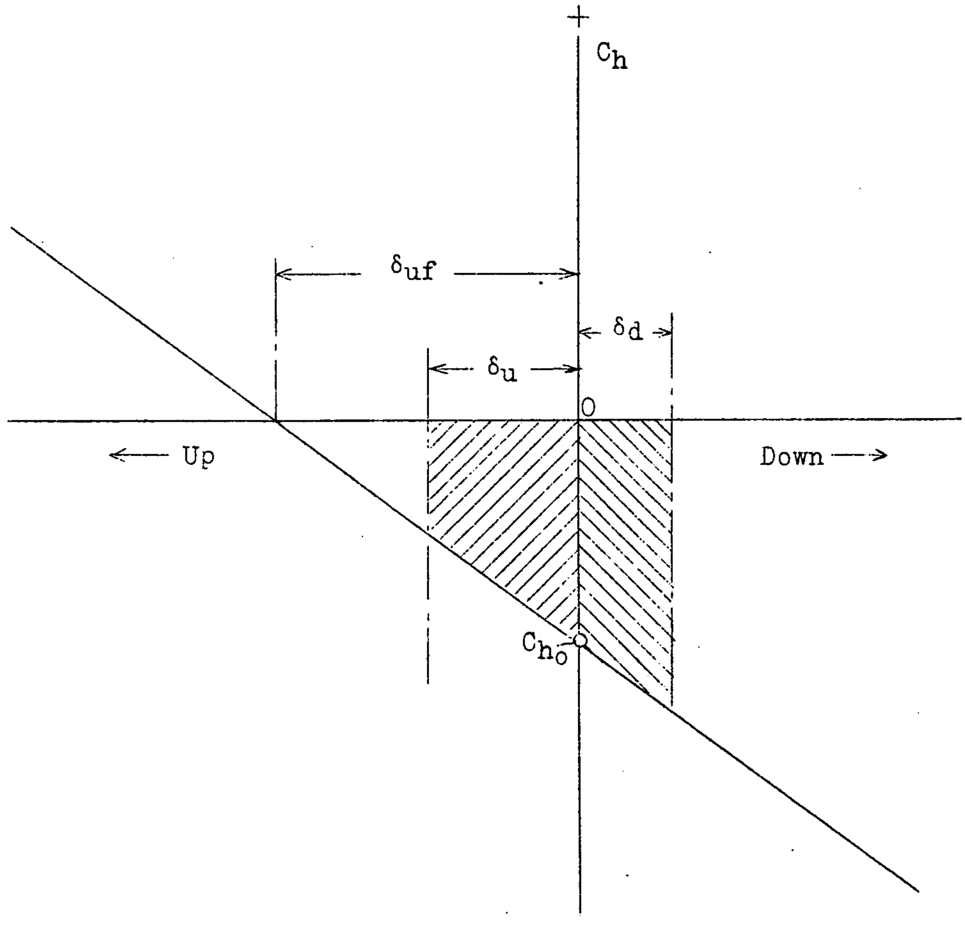


Figure 1.- Plot of aileron hinge-moment coefficient showing work of deflection.

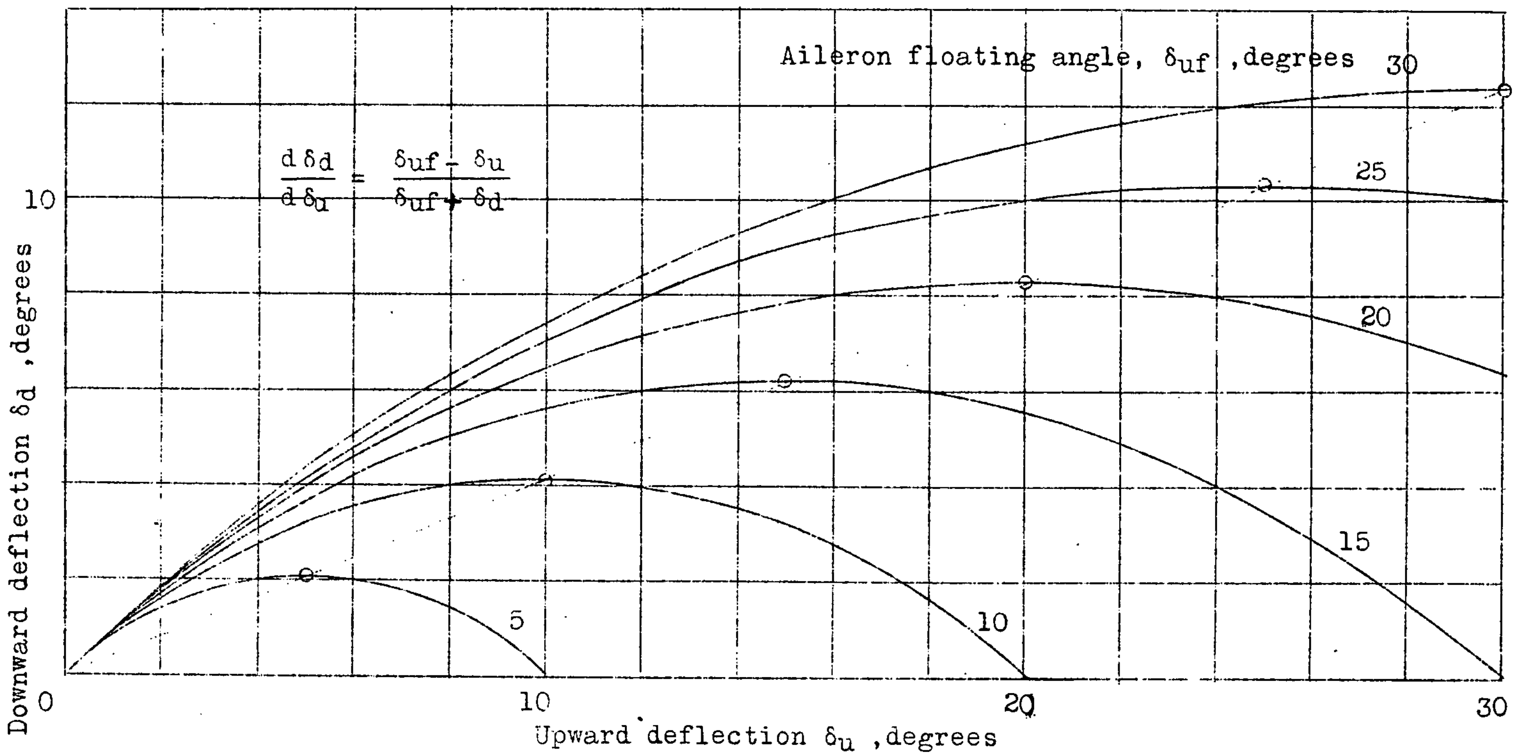


Figure 2.- Simultaneous positions of ailerons for which the work of deflection is zero.

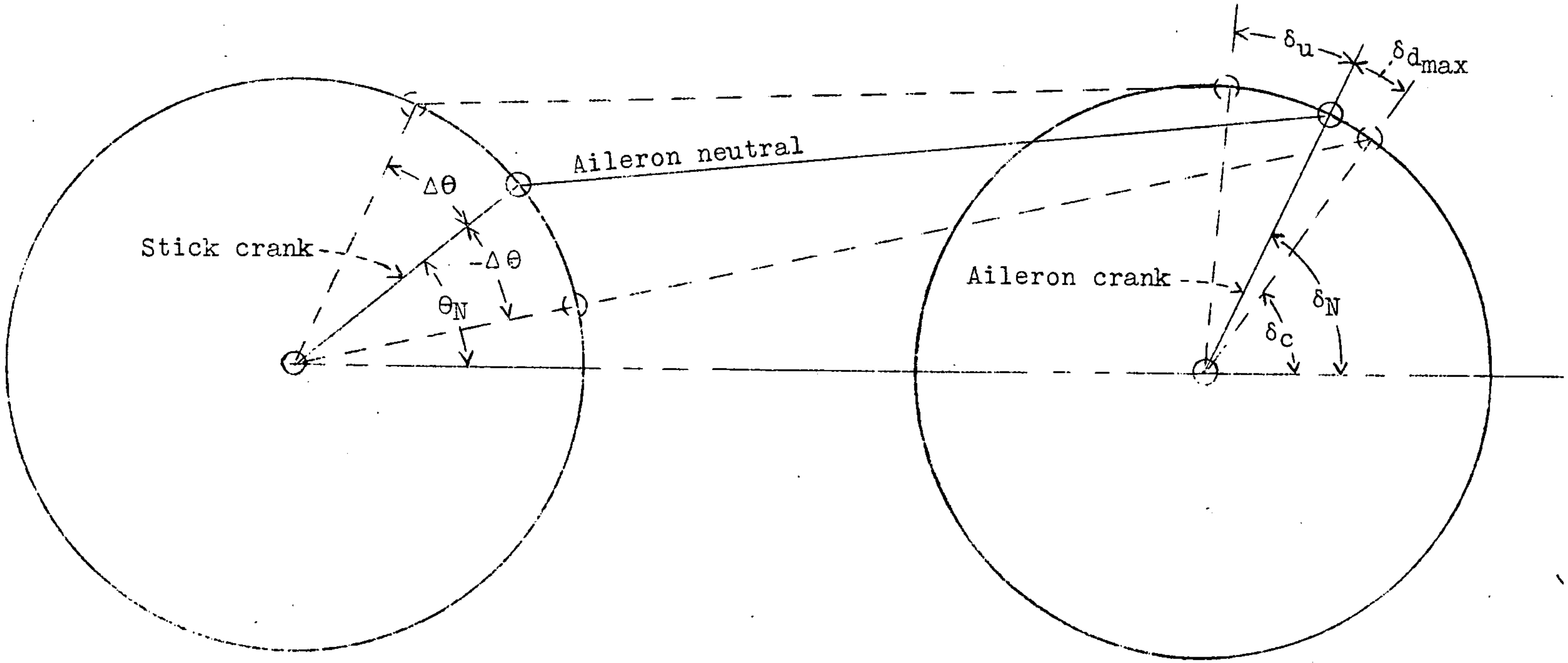
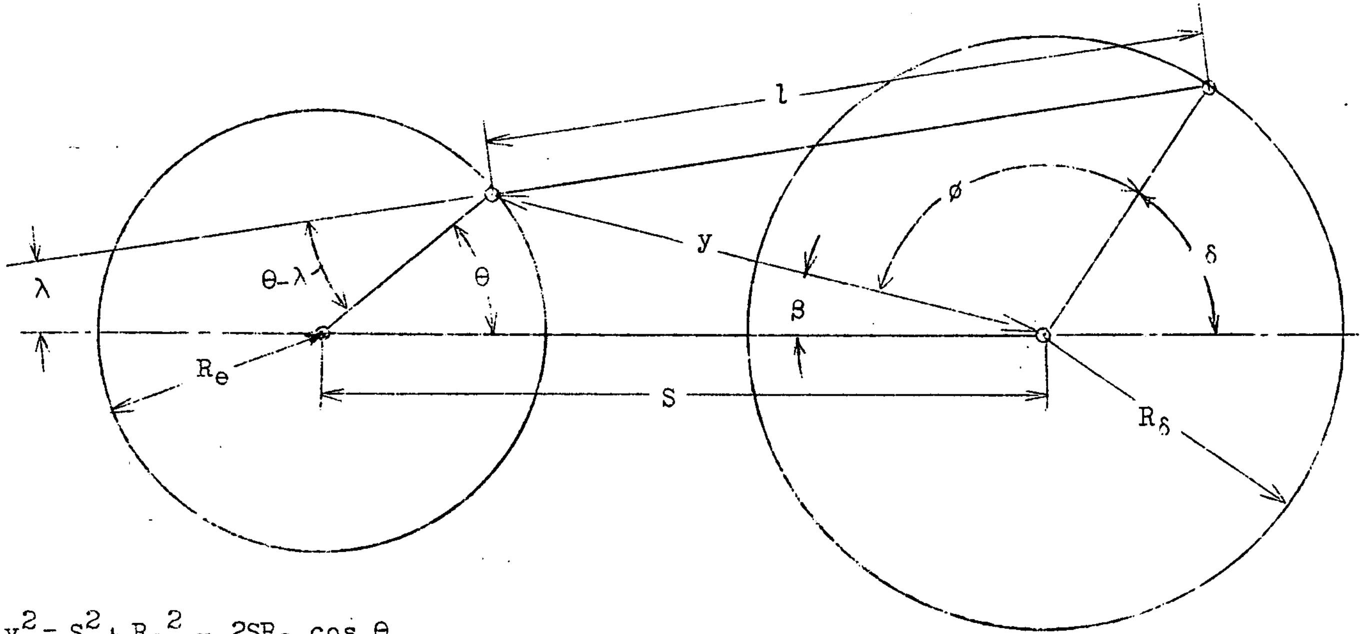


Figure 3.- Diagram representing simple differential linkage.



$$y^2 = S^2 + R_\theta^{\,2} - 2SR_\theta \cos\theta$$
$$\sin\beta = R_\theta \sin\theta / y$$
$$\cos\phi = (y^2 + R_\delta^{\,2} - l^2) / 2yR_\delta$$
$$\delta = 180^\circ - (\beta + \phi)$$
$$\sin\lambda = (R_\delta \sin\delta - R_\theta \sin\theta) / l$$
$$\frac{d\delta}{d\theta} = \frac{R_\theta}{R_\delta}\,\frac{\sin(\theta-\lambda)}{\sin(\delta-\lambda)}$$

Note: S, l, R_θ, R_δ are assumed to be known: θ is then varied as desired and the corresponding values of δ are found.

Figure 4.- Diagram and formulas for calculating the characteristics of a differential linkage.

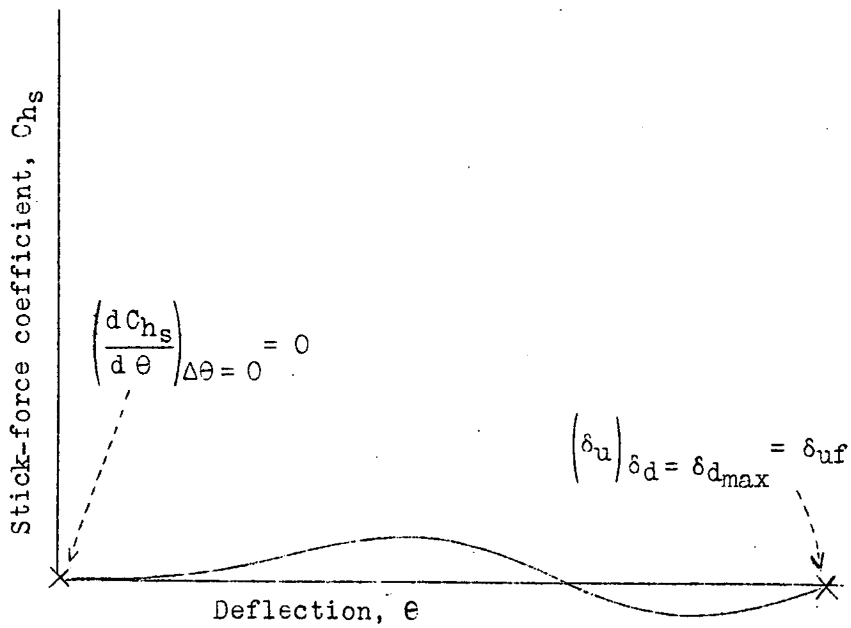
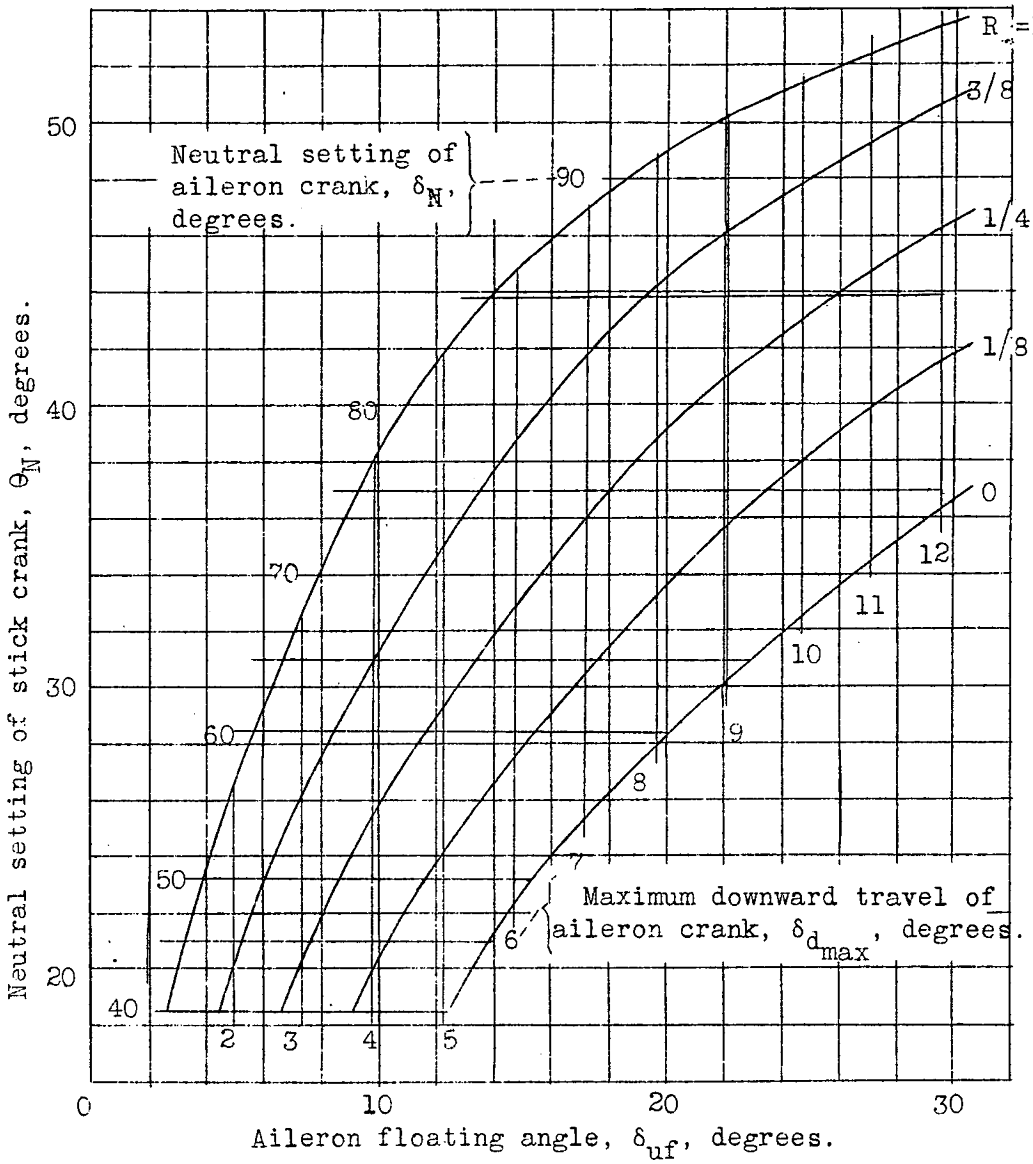


Figure 5.- Type of curve that satisfies simple criterions for minimum stick force.



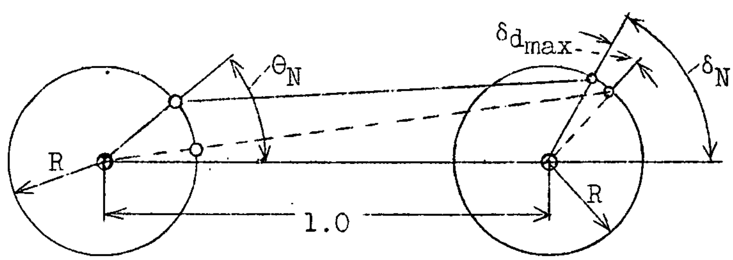


Figure 6.- Specifications of differentials that satisfy criterions for minimum stick force:

$$(dC_{h_s}/d\theta)_{\theta=\theta_N}' = 0; \qquad (\delta_u)_{\delta_d=\delta_{d_{max}}} = \delta_{uf}.$$

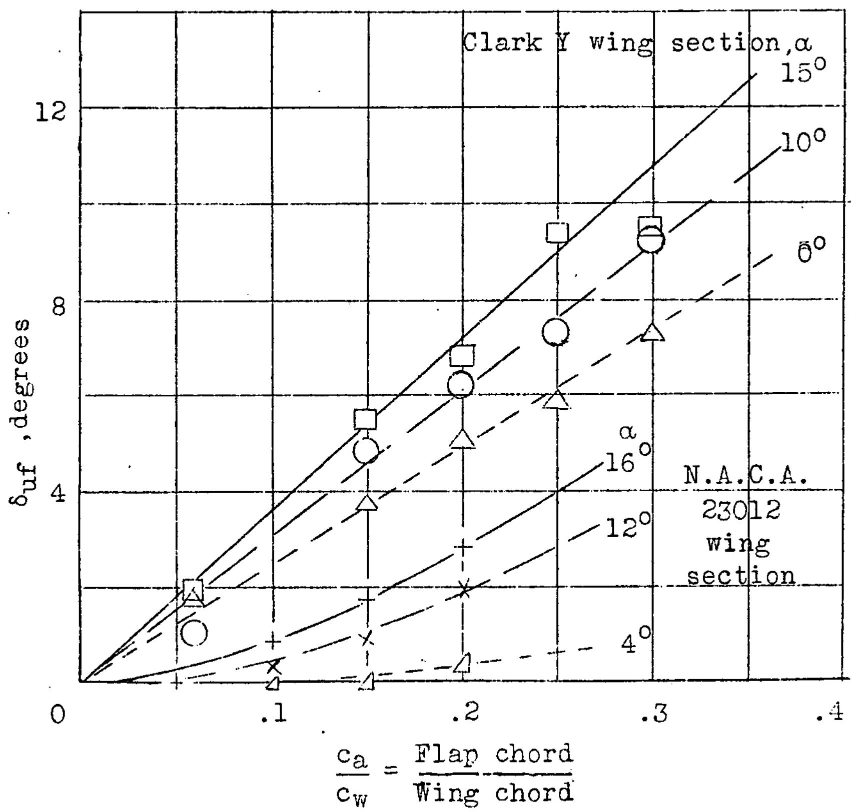
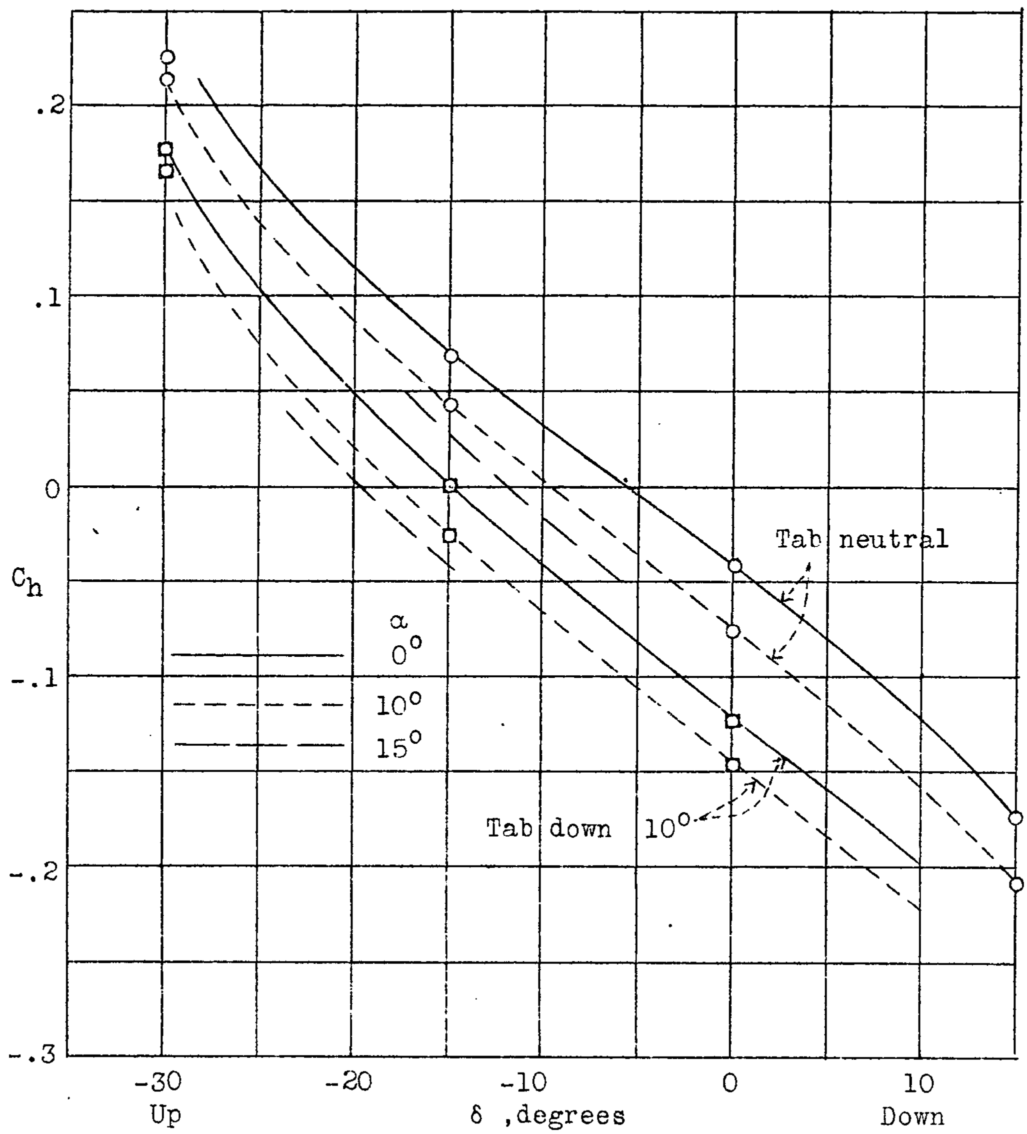


Figure 7.- Floating angles of flaps of different chords computed from pressure- distribution measurements.



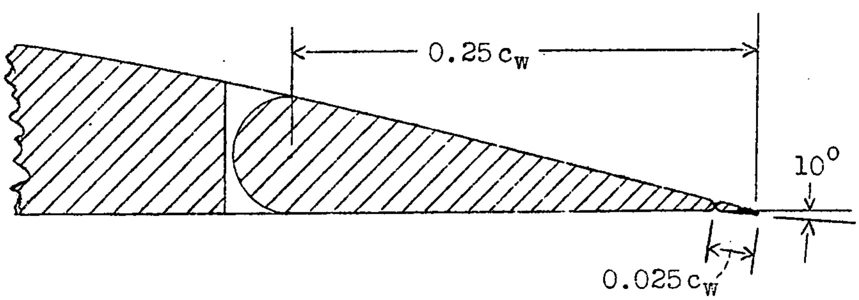


Figure 8.- Hinge-moment coefficients of aileron with tab. (reference 2).

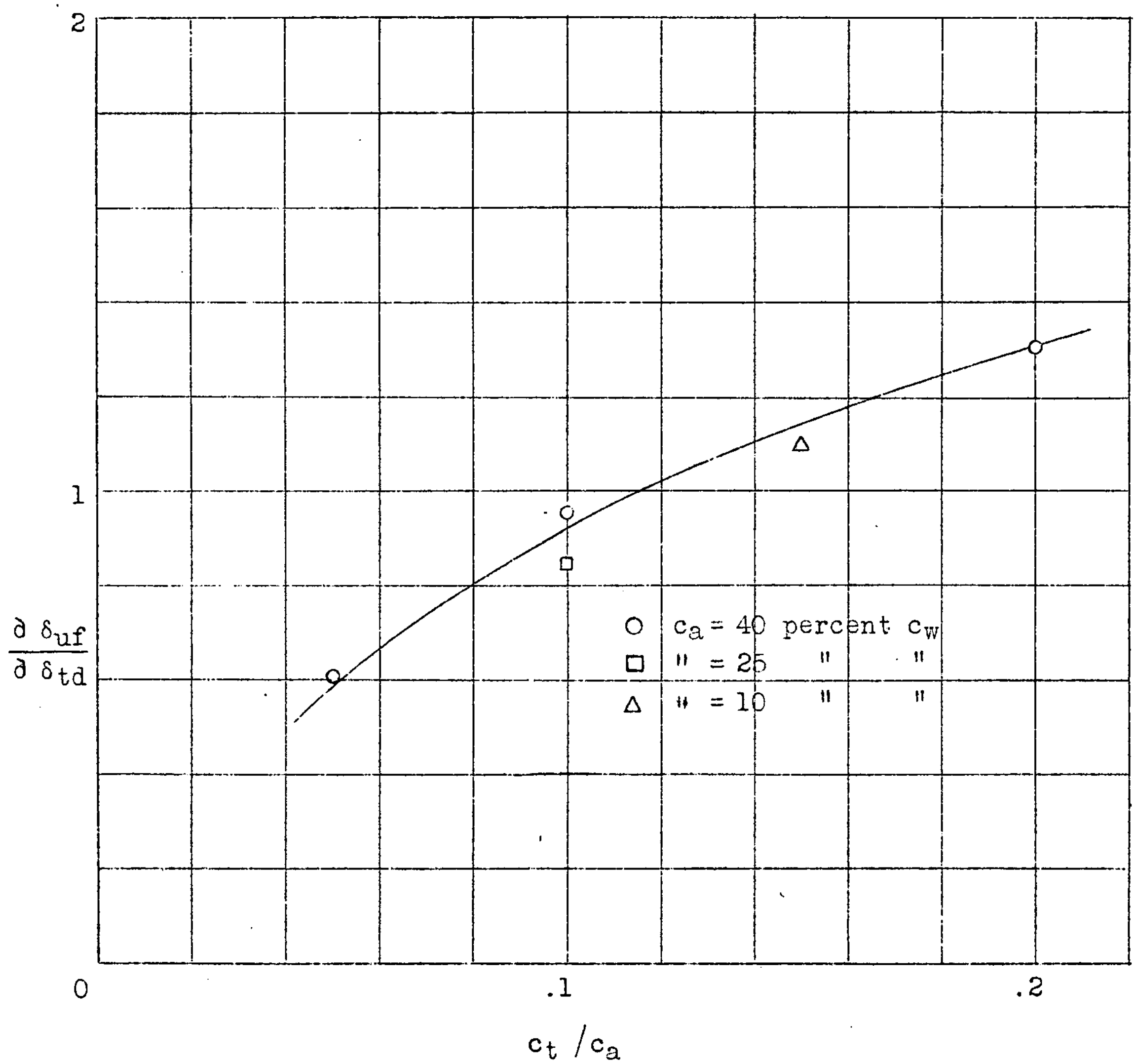


Figure 9.- Effect of tabs on aileron floating angles at small deflections ($\delta_t < 15^o$); 7 by 10 foot wind tunnel experiments.

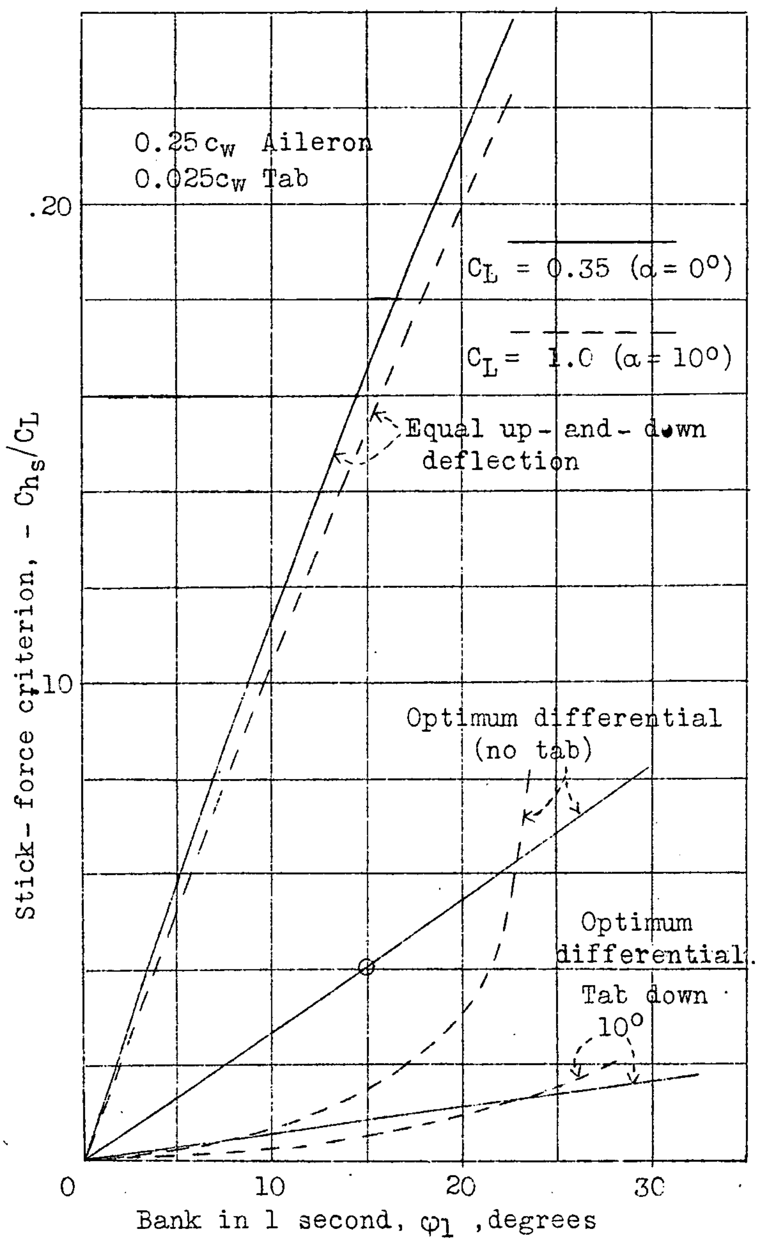


Figure 10.- Example showing reduction of stick force accomplished by suitable differential. The effect of a tab deflected downward to increase the floating angle is also shown.